

## 12 Gravitational Force Near the Surface of the Earth, First Brush with Newton's 2<sup>nd</sup> Law

*Some folks think that every object near the surface of the earth has an acceleration of  $9.8 \text{ m/s}^2$  downward relative to the surface of the earth. That just isn't so. In fact, as I look around the room in which I write this sentence, all the objects I see have zero acceleration relative to the surface of the earth. Only when it is in freefall, that is, only when nothing is touching or pushing or pulling on the object except for the gravitational field of the earth, will an object experience an acceleration of  $9.8 \text{ m/s}^2$  downward relative to the surface of the earth.*

### **Gravitational Force near the Surface of the Earth**

We all live in the invisible gravitational field of the earth. Mass is always accompanied by a surrounding gravitational field. Any object that has mass, including the earth, is surrounded by a gravitational field. The greater the mass of the object, the stronger the field is. The earth has a huge mass; hence, it creates a strong gravitational field in the region of space around it. The gravitational field is a force-per-mass at each and every point in the region around the object, always ready and able to exert a force on any particle that finds itself in the gravitational field. The earth's gravitational field exists everywhere around the earth, not only everywhere in the air, but out beyond the atmosphere in outer space, and inside the earth as well. The effect of the gravitational field is to exert a force on any particle, any "victim," that finds itself in the field. The force on the victim depends on both a property of the victim itself, namely its mass, and on a property of the point in space at which the particle finds itself, the force-per-mass of the gravitational field at that point. The force exerted on the victim by the gravitational field is just the mass of the victim times the force-per-mass value of the gravitational field at the location of the victim.

Hold a rock in the palm of your hand. You can feel that something is pulling the rock downward. It causes the rock to make a temporary indentation in the palm of your hand and you can tell that you have to press upward on the bottom of the rock to hold it up against that downward pull. The "something" is the field that we have been talking about. It is called the *gravitational field of the earth*. It has both magnitude and direction so we use a vector variable, the symbol  $\vec{g}$  to represent it. In general, the magnitude and the direction of a gravitational field both vary from point to point in the region of space where the gravitational field exists. The gravitational field of the earth, near the surface of the earth is however, to a very good approximation, much simpler than that. To a very good approximation, the gravitational field of the earth has the same value at all points near the surface of the earth, and it always points toward the center of the earth, a direction that we normally think of as downward. To a very good approximation,

$$\vec{g} = 9.80 \frac{\text{N}}{\text{kg}} \quad \text{downward} \quad (12-1)$$

at all points near the surface of the earth. The fact that the gravitational field is a force-per-mass at every point in space means that it must have units of force-per-mass. Indeed, the N (newton)

appearing in the value  $9.80 \frac{\text{N}}{\text{kg}}$  is the SI unit of force (how strong the push or pull on the object is) and the kg (kilogram) is the SI unit of mass, so the N/kg is indeed a unit of force-per-mass.

The gravitational force exerted on an object by the earth's gravitational field (or that of another planet when the object is near the surface of that other planet) is sometimes called the weight of the object. To stress that the gravitational force is a force that is being exerted on the object, rather than a property of the object itself, we will refer to it as the gravitational force. The gravitational force  $\vec{F}_g$  exerted on an object of mass  $m$  by the gravitational field of the earth is given by

$$\vec{F}_g = m\vec{g} \quad (12-2)$$

The product of a scalar and a vector is a new vector in the same direction as the original vector. Hence the earth's gravitational force is in the same direction as the gravitational field, namely downward, toward the center of the earth. The magnitude of the product of a scalar and a vector is the product of the absolute value of the scalar and the magnitude of the vector. [Recall that the magnitude of a vector is how big it is. A vector has both magnitude (how big) and direction (which way). So for instance, the magnitude of the force vector  $\vec{F} = 15 \text{ N downward}$ , is  $F = 15 \text{ newtons}$ .] Hence,

$$F_g = mg \quad (12-3)$$

relates the magnitude of the gravitational force to the magnitude of the gravitational field. The bottom line is that every object *near the surface* of the earth experiences a downward-directed gravitational force whose magnitude is given by  $F_g = mg$  where  $m$  is the mass of the object and

$g$  is  $9.80 \frac{\text{N}}{\text{kg}}$ .

### **When the Gravitational Force is the Only Force on an Object**

If there is a non-zero net force on an object, that object is experiencing acceleration in the same direction as that net force. How much acceleration depends on how big the net force is and on the mass of the object whose acceleration we are talking about, the object upon which the net force acts. In fact, the acceleration is directly proportional to the force. The constant of proportionality is the reciprocal of the mass of the object.

$$\vec{a} = \frac{1}{m} \sum \vec{F} \quad (12-4)^1$$

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<sup>1</sup> Equation 12-4 is known as Newton's 2<sup>nd</sup> Law. Newton's 2<sup>nd</sup> Law can also be written  $\sum \vec{F} = \frac{d\vec{p}}{dt}$  where  $\vec{p}$  is the momentum of the object. This latter equation is valid at all possible speeds, even at speeds close to the speed of light. To derive equation 12-4 from it, we use  $\vec{p} = m\vec{v}$  which is only valid for speeds small compared to the speed of light. Hence, equation 12-4 is only valid for speeds small compared to the speed of light ( $3.00 \times 10^8 \text{ m/s}$ ). Also,

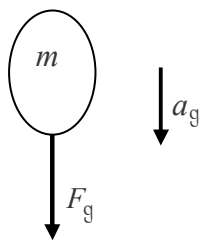
The expression  $\sum \vec{F}$  means “the sum of the forces acting on the object.” It is a vector sum. It is the net force acting on the object. The mass  $m$  is the inertia of the object, the object's inherent resistance to a change in its velocity. (Inherent means “of itself.”) Note that the factor  $\frac{1}{m}$  in equation 12-4:

$$\vec{a} = \frac{1}{m} \sum \vec{F}$$

means that the bigger the mass of the object, the smaller its acceleration will be, for a given net force. Equation 12-4 is a concise statement of a multitude of experimental results. It is referred to as “Newton's 2<sup>nd</sup> Law.” Here, we want to apply it to find the acceleration of an object in freefall near the surface of the earth.

Whenever you apply Newton's 2<sup>nd</sup> Law, you are required to draw a free body diagram of the object whose acceleration is under investigation. In a free body diagram, you depict the object (in our case it is an arbitrary object, let's think of it as a rock) free from all its surroundings, and then draw an arrow on it for each force acting on the object. Draw the arrow with the tail touching the object, and the arrow pointing in the direction of the force. Label the arrow with the symbol used to represent the magnitude of the force. Finally, draw an arrow near, but not touching, the object. Draw the arrow so that it points in the direction of the acceleration of the object and label it with a symbol chosen to represent the magnitude of the acceleration. Here we use the symbol  $a_g$  for the acceleration to remind us that it is the acceleration due to the earth's gravitational field  $\vec{g}$ .

***Free Body Diagram for an Object in Freefall near the Surface of the Earth***



The next step in applying Newton's 2<sup>nd</sup> Law is to write it down.

$$a_{\downarrow} = \frac{1}{m} \sum F_{\downarrow} \quad (12-5)$$

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the concept of force proves impractical on the atomic scale and smaller (distances less than about  $1 \times 10^{-9}$  m). Such small scales are the realm of quantum mechanics where energy and momentum still play a major role.

Note that equation 12-4:

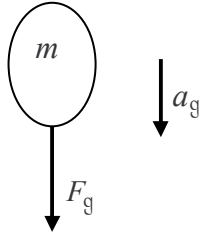
$$\vec{a} = \frac{1}{m} \sum \vec{F}$$

is a vector equation. As such it can be considered to be three equations in one—one equation for each of a total of three possible mutually-orthogonal (meaning perpendicular to each other) coordinate directions in space. In the case at hand, all the vectors, (hey, there are only two, the gravitational force vector and the acceleration vector) are parallel to one and the same line, namely the vertical, so we only need one of the equations. In equation 12-5,

$$a_{\downarrow} = \frac{1}{m} \sum F_{\downarrow}$$

we use arrows as subscripts—the arrow shaft alignment specifies the line along which we are summing the forces and the arrowhead specifies the direction along that line that we choose to call the positive direction. In the case at hand, referring to equation 12-5, we note that the shafts of the arrows are vertical, meaning that we are summing forces along the vertical and that we are dealing with an acceleration along the vertical. Also in equation 12-5, we note that the arrowheads are pointing downward meaning that I have chosen to call downward the positive direction, which, by default, means that upward is the negative direction. (I chose to call downward positive because both of the vectors in the free body diagram are downward.)

Next we replace  $a_{\downarrow}$  with what it is in the free body diagram,



namely  $a_g$ , and we replace  $\sum F_{\downarrow}$  with the sum of the vertical forces in the free body diagram, counting downward forces as positive contributions to the sum, and upward forces as negative contributions to the sum. This is an easy substitution in the case at hand because there is only one force on the free body diagram, namely the gravitational force, the downward force of magnitude  $F_g$ . The result of our substitutions is:

$$a_g = \frac{1}{m} F_g \quad (12-6)$$

The  $F_g$  in equation 12-6 is the magnitude of the gravitational force, that force which you already read about at the start of this chapter. It is given in terms of the mass of the object and the magnitude of the earth's gravitational field  $g$  by equation 12-3,  $F_g = mg$ . Replacing the  $F_g$  in

equation 12-6,  $a_g = \frac{1}{m} F_g$ , with the  $mg$  to which it is equivalent we have

$$a_g = \frac{1}{m} mg \quad (12-7)$$

Now the  $m$  that appears in the fraction  $\frac{1}{m}$  is the inertia of the object. It is the amount of inherent resistance that the object has to a change in its velocity and is a measure of the total amount of material making up the object. The  $m$  appearing in the  $mg$  part of the expression (equation 12-7) is the gravitational mass of the object, the quantity that, in concert with the gravitational field at the location of the object determines the force on the object. It is also a measure of the total amount of material making up the object. As it turns out, the inertial mass and the gravitational mass of the same object are identical (which is why we use one and the same symbol  $m$  for each) and, in equation 12-7, they cancel. Thus,

$$a_g = g \quad (12-8)$$

Now  $g$  is the magnitude of the earth's gravitational field vector at the location of the object.  $g = 9.80 \text{ N/kg}$  and  $a_g$ , being an acceleration has to have units of acceleration, namely,  $\text{m/s}^2$ .

Fortunately a newton is a  $\frac{\text{kg} \cdot \text{m}}{\text{s}^2}$  so the units of  $g$ , namely  $\text{N/kg}$ , do indeed work out to be  $\frac{\text{m}}{\text{s}^2}$ .

Thus

$$a_g = 9.80 \frac{\text{m}}{\text{s}^2} \quad (12-9)$$

Now this is “wild”! The acceleration of an object in freefall does not depend on its mass. You saw the masses cancel. The same thing that makes an object heavy makes it “sluggish.”

### ***One-Dimensional Free-Fall, a.k.a., One-Dimensional Projectile Motion***

If you throw an object straight up, or simply release it from rest, or throw it straight down; assuming that the force of air resistance is negligibly small compared to the gravitational force: the object will be in freefall from the instant it loses contact with your hand until the last instant before it hits the ground (or whatever it does eventually hit), and the object will travel along a straight line path with a constant acceleration of  $9.80 \frac{\text{m}}{\text{s}^2}$  downward.

Consider the case in which the object is thrown straight up. The whole time it is in freefall, the object experiences an acceleration of  $9.80 \frac{\text{m}}{\text{s}^2}$  downward. While the object is on the way up, the downward acceleration means that the object is slowing down. At the top of its motion, when the velocity changes from being an upward velocity to being a downward velocity, and hence, for an instant is zero, the downward acceleration means that the velocity is changing from zero to a non-zero downward velocity. And on the way down, the downward acceleration means that the velocity is increasing in the downward direction.