14 Newton's Laws #1: Using Free Body Diagrams

If you throw a rock upward in the presence of another person, and you ask that other person what keeps the rock going upward, after it leaves your hand but before it reaches its greatest height, that person may incorrectly tell you that the force of the person's hand keeps it going. This illustrates the common misconception that force is something that is given to the rock by the hand and that the rock "has" while it is in the air. It is not. A force is all about something that is being done to an object. We have defined a force to be an ongoing push or a pull. It is something that an object can be a victim to, it is never something that an object has. While the force is acting on the object, the motion of the object is consistent with the fact that the force is acting on the object. Once the force is no longer acting on the object, there is no such force, and the motion of the object is consistent with the fact that the force is absent. (As revealed in this chapter, the correct answer to the question about what keeps the rock going upward, is, "Nothing." Continuing to go upward is what it does all by itself if it is already going upward. You don't need anything to make it keep doing that. In fact, the only reason the rock does not continue to go upward forever is because there is a downward force on it. When there is a downward force and only a downward force on an object, that object is experiencing a downward acceleration. This means that the upward-moving rock slows down, then reverses its direction of motion and moves downward ever faster.)

Imagine that the stars are fixed in space so that the distance between one star and another never changes. (They are not fixed. The stars are moving relative to each other.) Now imagine that you create a Cartesian coordinate system; a set of three mutually orthogonal axes that you label x, y, and z. Your Cartesian coordinate system is a reference frame. Now as long as your reference frame is not rotating and is either fixed or moving at a constant velocity relative to the (fictitious) fixed stars, then your reference frame is an *inertial reference frame*. Note that velocity has both magnitude and direction and when we stipulate that the velocity of your reference frame must be constant in order for it to be an inertial reference frame, we aren't just saying that the magnitude has to be constant but that the direction has to be constant as well. The magnitude of the velocity is the speed. So, for the magnitude of the velocity to be constant, the speed must be constant. For the direction to be constant, the reference frame must move along a straight line path. So an inertial reference frame is one that is either fixed or moving at a constant speed along a straight line path, relative to the (fictitious) fixed stars.

The concept of an inertial reference frame is important in the study of physics because it is in inertial reference frames that the laws of motion known as Newton's Laws of Motion apply. Here are Newton's three laws of motion, observed to be adhered to by any particle of matter in an inertial reference frame:

I. If there is no net force acting on a particle, then the velocity of that particle is not changing.

II. If there is a net force on a particle, then that particle is experiencing an acceleration that is directly proportional to the force, with the constant of proportionality being the reciprocal of the mass of the particle.

III. Anytime one object is exerting a force on a second object, the second object is exerting an equal but opposite force back on the first object.

Discussion of Newton's 1st Law

Despite the name, it was actually Galileo that came up with the first law. He let a ball roll down a ramp with another ramp facing the other way in front of it so that, after it rolled down one ramp, the ball would roll up the other. He noted that the ball rolled up the second ramp, slowing steadily until it reached the same elevation as the one from which the ball was originally released from rest. He then repeatedly reduced the angle that the second ramp made with the horizontal and released the ball from rest from the original position for each new inclination of the second ramp. The smaller the angle, the more slowly the speed of the ball was reduced on the way up the second ramp and the farther it had to travel along the surface of the second ramp before arriving at its starting elevation. When he finally set the angle to zero, the ball did not appear to slow down at all on the second ramp. He didn't have an infinitely long ramp, but he induced that if he did, with the second ramp horizontal, the ball would keep on rolling forever, never slowing down because no matter how far it rolled, it would never gain any elevation, so it would never get up to the starting elevation. His conclusion was that if an object was moving, then if nothing interfered with its motion it would keep on moving at the same speed in the same direction. So what keeps it going? The answer is "nothing." That is the whole point. An object doesn't need anything to keep it going. If it is already moving, going at a constant velocity is what it does as long as there is no net force acting on it. In fact, it takes a *force* to *change* the velocity of an object.

It's not hard to see why it took a huge chunk of human history for someone to realize that if there is no net force on a moving object, it will keep moving at a constant velocity, because the thing is, where we live, on the surface of the Earth, there is inevitably a net force on a moving object. You throw something up and the Earth pulls downward on it the whole time the object is in flight. It's not going to keep traveling in a straight line upward, not with the Earth pulling on it. Even if you try sliding something across the smooth surface of a frozen pond where the downward pull of the Earth's gravitational field is cancelled by the ice pressing up on the object, you find that the object slows down because of a frictional force pushing on the object in the direction opposite that of the object's velocity and indeed a force of air resistance doing the same thing. In the presence of these ubiquitous forces, it took humankind a long time to realize that if there were no forces, an object in motion would stay in motion along a straight line path, at constant speed, and that an object at rest would stay at rest.

Discussion of Newton's 2nd Law

Galileo induced something else of interest from his ball-on-the-ramp experiments by focusing his attention on the first ramp discussed above. Observation of a ball released from rest revealed to him that the ball steadily sped up on the way down the ramp. Try it. As long as you don't make the ramp too steep, you can *see* that the ball doesn't just roll down the ramp at some fixed speed, it accelerates the whole way down. Galileo further noted that the steeper the ramp was, the faster the ball would speed up on the way down. He did trial after trial, starting with a slightly inclined plane and gradually making it steeper and steeper. Each time he made it steeper, the ball would, on the way down the ramp, speed up faster than it did before, until the ramp got so steep that he could no longer see that it was speeding up on the way down the ramp—it was simply happening too fast to be observed. But Galileo induced that, as he continued to make the ramp steeper, the same thing was happening. That is that the ball's speed was still increasing on its way down the ramp and the greater the angle, the faster the ball would speed up. In fact, he induced that if he increased the steepness to the ultimate angle, 90°, that the ball would speed up the whole way down the ramp faster than it would at any smaller angle but that it would still speed up on the way down. Now, when the ramp is tilted at 90°, the ball is actually falling as opposed to rolling down the ramp, so Galileo's conclusion was that when you drop an object (for which air resistance is negligible), what happens is that the object speeds up the whole way down, until it hits the Earth.

Galileo thus did quite a bit to set the stage for Sir Isaac Newton, who was born the same year that Galileo died.

It was Newton who recognized the relationship between force and motion. He is the one that realized that the link was between force and acceleration, more specifically, that whenever an object is experiencing a net force, that object is experiencing an acceleration in the same direction as the force. Now, some objects are more sensitive to force than other objects—we can say that every object comes with its own sensitivity factor such that the greater the sensitivity factor, the greater the acceleration of the object for a given force. The sensitivity factor is the reciprocal of the mass of the object, so we can write that

$$\vec{a} = \frac{1}{m}\sum \vec{\mathbf{F}} \tag{14-1}$$

where $\vec{a}$ is the acceleration of the object, m is the mass of the object, and $\sum \vec{\mathbf{F}}$ is the vector sum of all the forces acting on the object, that is to say that $\sum \vec{\mathbf{F}}$ is the net force acting on the object.

Discussion of Newton's Third Law

In realizing that whenever one object is in the act of exerting a force on a second object, the second object is always in the act of exerting an equal and opposite force back on the first object, Newton was recognizing an aspect of nature that, on the surface, seems quite simple and straightforward, but quickly leads to conclusions that, however correct they may be, and indeed they are correct, are quite counterintuitive. Newton's 3rd law is a statement of the fact that any force whatsoever is just one half of an interaction where an *interaction* in this sense is the mutual pushing or pulling that quite often occurs when one object is in the vicinity of another.

In some cases, where the effect is obvious, the validity of Newton's third law is fairly evident. For instance if two people who have the same mass are on roller skates and are facing each other and one pushes the other, we see that both skaters go rolling backward, away from each other. It might at first be hard to accept the fact that the second skater is pushing back on the hands of the first skater, but we can tell that the skater that we think of as the pusher, must also be a "pushee," because we can see that she experiences a backward acceleration. In fact, while the pushing is taking place, the force exerted on her must be just as great as the force she exerts on the other skater because we see that her final backward speed is just as great as that of the other (same-mass) skater.

But how about those cases where the effect of at least one of the forces in the interaction pair is not at all evident? Suppose for instance that you have a broom leaning up against a slippery wall. Aside from our knowledge of Newton's laws, how can we convince ourselves that the broom is pressing against the wall, that is, that the broom is continually exerting a force on the wall; and; how can we convince ourselves that the wall is exerting a force back on the broom? One way to convince yourself is to let your hand play the role of the wall. Move the broom and put your hand in the place of the wall so that the broom is leaning against the palm of your hand at the same angle that it was against the wall with the palm of your hand facing directly toward the tip of the handle. You can feel the tip of the handle pressing against the palm of your hand. In fact, you can see the indentation that the tip of the broom handle makes in your hand. You can feel the force of the broom handle on your hand and you can induce that when the wall is where your hand is, relative to the broom, the broom handle must be pressing on the wall with the same force.

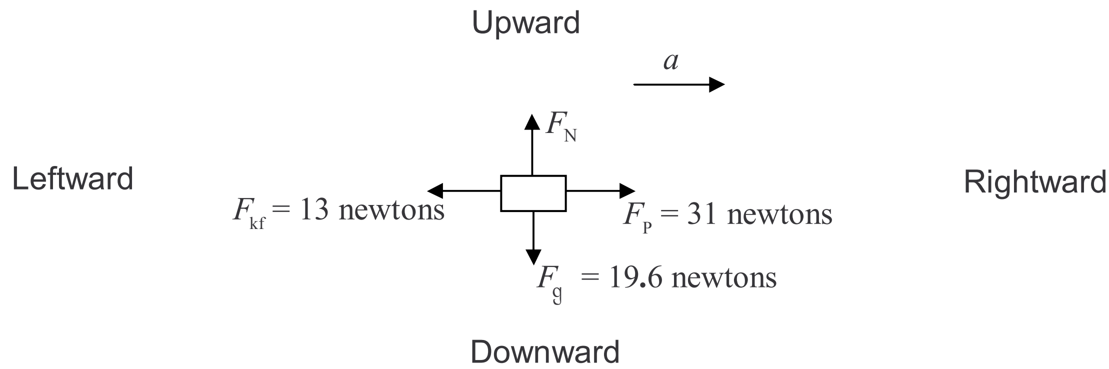
How about this business of the wall exerting a force on (pushing on) the tip of the broom handle? Again, with your hand playing the role of the wall, quickly move your hand out of the way. The broom, of course, falls down. Before moving your hand, you must have been applying a force on the broom or else the broom would have fallen down then. You might argue that your hand wasn't necessarily applying a force but rather that your hand was just "in the way." Well I'm here to tell you that "being in the way" is all about applying a force. When the broom is leaning up against the wall, the fact that the broom does not fall over means that the wall is exerting a force on the broom that cancels the other forces so that they don't make the broom fall over. In fact, if the wall was not strong enough to exert such a force, the wall would break. Still, it would be nice to get a visceral sense of the force exerted on the broom by the wall. Let your hand play the role of the wall, but this time, let the broom lean against your pinky, near the tip of your finger. To keep the broom in the same orientation as it was when it was leaning against the wall, you can feel that you have to exert a force on the tip of the broom handle. In fact, if you increase this force a little bit, the broom handle tilts more upward, and if you decrease it, it tilts more downward. Again, you can feel that you are pushing on the tip of the broom handle when you are causing the broom handle to remain stationary at the same orientation it had when it was leaning against the wall, and you can induce that when the wall is where your hand is, relative to the broom, the wall must be pressing on the broom handle with the same force. Note that the direction in which the wall is pushing on the broom is away from the wall at right angles to the wall. Such a force is exerted on any object that is in contact with a solid surface. This contact force exerted by a solid surface on an object in contact with that surface is called a "normal force" because the force is perpendicular to the surface and the word "normal" means perpendicular.

Using Free Body Diagrams

The key to the successful solution of a Newton's 2nd Law problem is to draw a good free body diagram of the object whose motion is under study and then to use that free body diagram to expand Newton's 2nd Law, that is, to replace the $\sum \vec{\mathbf{F}}$ with an the actual term-by-term sum of the forces. Note that Newton's 2nd Law $\vec{a} = \frac{1}{m}\sum \vec{\mathbf{F}}$ is a vector equation and hence, in the most general case (3 dimensions) is actually three scalar equations in one, one for each of the three possible mutually orthogonal directions in space. (A scalar is a number. Something that has magnitude only, as opposed to a vector which has magnitude and direction.) In your physics course, you will typically be dealing with forces that all lie in the same plane, and hence, you will typically get two equations from $\vec{a} = \frac{1}{m}\sum \vec{\mathbf{F}}$.

Regarding the Free Body Diagrams: The hard part is creating them from a description of the physical process under consideration; the easy part is using them. In what little remains of this chapter, we will focus on the easy part: Given a Free Body Diagram, use it to find an unknown force or unknown forces, and/or use it to find the acceleration of the object.

For example, given the free body diagram



for an object of mass 2.00 kg, find the magnitude of the normal force F_N and find the magnitude of the acceleration a. (Note that we define the symbols that we use to represent the components of forces and the component of the acceleration, *in the free body diagram*. We do this by drawing an arrow whose shaft represents a line along which the force lies, and whose arrowhead we define to be the positive direction for that force component, and then labeling the arrow with our chosen symbol. A negative value for a symbol thus defined, simply means that the corresponding force or acceleration is in the direction opposite to the direction in which the arrow is pointing.

Solution: Note that the acceleration and all of the forces lie along one or the other of two imaginary lines (one of which is horizontal and the other of which is vertical) that are perpendicular to each other. The acceleration along one line is independent of any forces perpendicular to that line so we can consider one line at a time. Let's deal with the horizontal line first. We write Newton's 2nd Law for the horizontal line as

$$a_{\rightarrow} = \frac{1}{m}\sum F_{\rightarrow} \tag{14-2}$$

in which the shafts of the arrows indicate the line along which we are summing forces (the shafts in equation 14-2 are horizontal so we must be summing forces along the horizontal) and the arrowhead indicates which direction we consider to be the positive direction (any force in the opposite direction enters the sum with a minus sign).

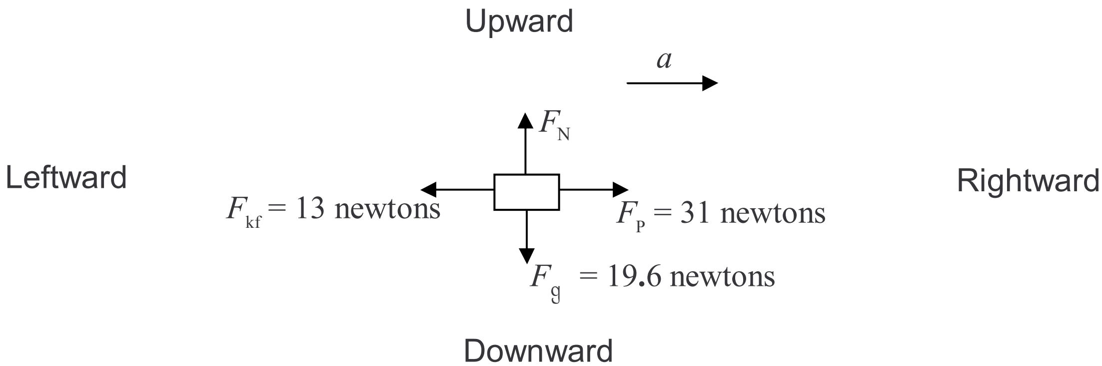
The next step is to replace $a_{\rightarrow}$ with the symbol that we have used in the diagram to represent the rightward acceleration and the $\sum F_{\rightarrow}$ with an actual term-by-term sum of the forces which includes only horizontal forces and in which rightward forces enter with a "+" and leftward forces enter with a "−". This yields:

$$a = \frac{1}{m}(F_P - F_{kf})$$

Substituting values with units and evaluating gives:

$$a = \frac{1}{2.00\text{ kg}}(31\text{ N} - 13\text{ N}) = 9.0\frac{\text{m}}{\text{s}^2}$$

Now we turn our attention to the vertical direction. For your convenience, the free body diagram is replicated here:



Again we start with Newton's 2nd Law, this time written for the vertical direction:

$$a_{\downarrow} = \frac{1}{m}\sum F_{\downarrow}$$

We replace $a_{\downarrow}$ with what it is and we replace $\sum F_{\downarrow}$ with the term-by-term sum of the forces with a "+" for downward forces and a "−" for upward forces. Note that the only a in the free body diagram is horizontal. Whoever came up with that free body diagram is telling us that there is no acceleration in the vertical direction, that is, that $a_{\downarrow} = 0$. Thus:

$$0 = \frac{1}{m}(F_g - F_N)$$

Solving this for F_N yields

$$F_N = F_g$$

Substituting values with units results in a final answer of:

$$\boxed{F_N = 19.6 \text{ newtons.}}$$