

## 32 Beats, The Doppler Effect

*Some people get mixed up about the Doppler Effect. They think it's about position rather than about velocity. (It is really about velocity.) If a single frequency sound source is coming at you at constant speed, the pitch (frequency) you hear **is** higher than the frequency of the source. How much higher depends on how fast the source is coming at you. Folks make the mistake of thinking that the pitch gets higher as the source approaches the receiver. No. That would be the case if the frequency depended on how close the source was to the receiver. It doesn't. The frequency stays the same. The Doppler Effect is about velocity, **not** position. The whole time the source is moving straight at you, it will sound like it has one single unchanging pitch that is higher than the frequency of the source. **Now duck!** Once the object passes your position and it is heading away from you it will have one single unchanging pitch that is lower than the frequency of the source.*

### Beats

Consider two sound sources, in the vicinity of each other, each producing sound waves at its own single frequency. Any point in the air-filled region of space around the sources will receive sound waves from both the sources. The amplitude of the sound at any position in space will be the amplitude of the sum of the displacements of the two waves at that point. This amplitude will vary because the interference will alternate between constructive interference and destructive interference. Suppose the two frequencies do not differ by much. Consider the displacements at a particular point in space. Let's start at an instant when two sound wave crests are arriving at that point, one from each source. At that instant the waves are interfering constructively, resulting in a large total amplitude. If your ear were at that location, you would find the sound relatively loud. Let's mark the passage of time by means of the shorter period, the period of the higher-frequency waves. One period after the instant just discussed, the next crest (call it the second crest) from the higher-frequency source is at the point in question, but the peak of the next crest from the lower-frequency source is not there yet. Rather than a crest interfering with a crest, we have a crest interfering with an intermediate-displacement part of the wave. The interference is still constructive but not to the degree that it was. When the third crest from the higher-frequency source arrives, the corresponding crest from the lower-frequency source is even farther behind. Eventually, a crest from the higher-frequency source is arriving at the point in question at the same time as a trough from the lower-frequency source. At that instant in time, the interference is as destructive as it gets. If your ear were at the point in question, you would find the sound to be inaudible or of very low volume. Then the trough from the lower-frequency source starts "falling behind" until, eventually a crest from the higher-frequency source is interfering with the crest preceding the corresponding crest from the lower-frequency source and the interference is again as constructive as possible.

To a person whose ear is at a location at which waves from both sources exist, the sound gets loud, soft, loud, soft, etc. The frequency with which the loudness pattern repeats itself is called the beat frequency. Experimentally, we can determine the beat frequency by timing how long it takes for the sound to get loud  $N$  times and then dividing that time by  $N$  (where  $N$  is an arbitrary

integer chosen by the experimenter—the bigger the  $N$  the more precise the result). This gives the beat period. Taking the reciprocal of the beat period yields the beat frequency.

The beat frequency is to be contrasted with the ordinary frequency of the waves. In sound, we hear the beat frequency as the rate at which the loudness of the sound varies whereas we hear the ordinary frequency of the waves as the pitch of the sound.

### **Derivation of the Beat Frequency Formula**

Consider sound from two different sources impinging on one point, call it point P, in air-occupied space. Assume that one source has a shorter period  $T_{\text{SHORT}}$  and hence a higher frequency  $f_{\text{HIGH}}$  than the other (which has period and frequency  $T_{\text{LONG}}$  and  $f_{\text{LOW}}$  respectively). The plan here is to express the beat frequency in terms of the frequencies of the sources—we get there by relating the periods to each other. As in our conceptual discussion, let's start at an instant when a crest from each source is at point P. When, after an amount of time  $T_{\text{SHORT}}$  passes, the next crest from the shorter-period source arrives, the corresponding crest from the longer-period source won't arrive for an amount of time  $\Delta T = T_{\text{LONG}} - T_{\text{SHORT}}$ . In fact, with the arrival of each successive short-period crest, the corresponding long-period crest is another  $\Delta T$  behind. Eventually, after some number  $n$  of short periods, the long-period crest will arrive a full long period  $T_{\text{LONG}}$  after the corresponding short-period crest arrives.

$$n \Delta T = T_{\text{LONG}} \quad (32-1)$$

This means that as the short-period crest arrives, the long-period crest that precedes the corresponding long-period crest is arriving. This results in constructive interference (loud sound). The time it takes, starting when the interference is maximally constructive, for the interference to again become maximally constructive is the beat period

$$T_{\text{BEAT}} = n T_{\text{SHORT}} \quad (32-2)$$

Let's use equation 32-1 to eliminate the  $n$  in this expression. Solving equation 32-1 for  $n$  we find that

$$n = \frac{T_{\text{LONG}}}{\Delta T}$$

Substituting this into equation 32-2 yields

$$T_{\text{BEAT}} = \frac{T_{\text{LONG}}}{\Delta T} T_{\text{SHORT}}$$

$\Delta T$  is just  $T_{\text{LONG}} - T_{\text{SHORT}}$  so

$$T_{\text{BEAT}} = \frac{T_{\text{LONG}}}{T_{\text{LONG}} - T_{\text{SHORT}}} T_{\text{SHORT}}$$

$$T_{\text{BEAT}} = \frac{T_{\text{LONG}} T_{\text{SHORT}}}{T_{\text{LONG}} - T_{\text{SHORT}}}$$

Dividing top and bottom by the product  $T_{\text{LONG}} T_{\text{SHORT}}$  yields

$$T_{\text{BEAT}} = \frac{1}{\frac{1}{T_{\text{SHORT}}} - \frac{1}{T_{\text{LONG}}}}$$

Taking the reciprocal of both sides results in

$$\frac{1}{T_{\text{BEAT}}} = \frac{1}{T_{\text{SHORT}}} - \frac{1}{T_{\text{LONG}}}$$

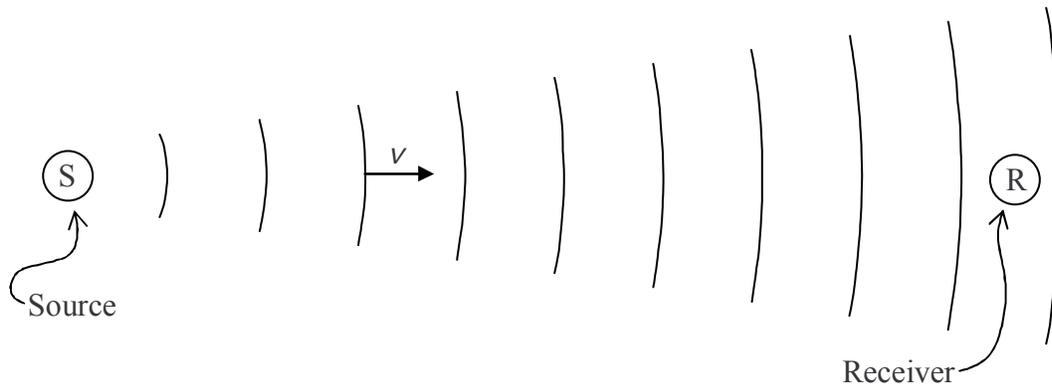
Now we use the frequency-period relation  $f = \frac{1}{T}$  to replace each reciprocal period with its corresponding frequency. This yields:

$$f_{\text{BEAT}} = f_{\text{HIGH}} - f_{\text{LOW}} \quad (32-3)$$

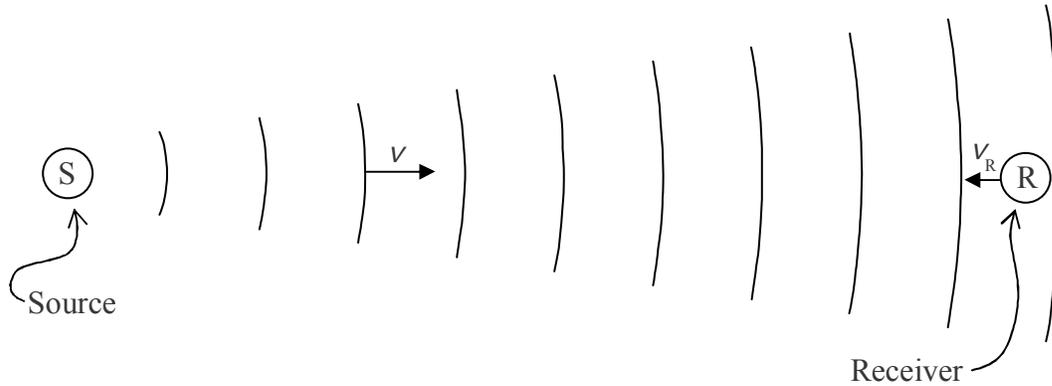
for the beat frequency in terms of the frequencies of the two sources.

### The Doppler Effect

Consider a single-frequency sound source and a receiver. The source is something oscillating. It produces sound waves. They travel through air, at speed  $v$ , the speed of sound in air, to the receiver and cause some part of the receiver to oscillate. (For instance, if the receiver is your ear, the sound waves cause your eardrum to oscillate.) If the receiver and the source are at rest relative to the air, then the received frequency is the same as the source frequency.



But if the source is moving toward or away from the receiver, and/or the receiver is moving toward or away from the source, the received frequency will be different from the source frequency. Suppose for instance, the receiver is moving toward the source with speed  $v_R$ .



The receiver meets wave crests more frequently than it would if it were still. Starting at an instant when a wavefront is at the receiver, the receiver and the next wavefront are coming together at the rate  $v + v_R$  (where  $v$  is the speed of sound in air). The distance between the wavefronts is just the wavelength  $\lambda$  which is related to the source frequency  $f$  by  $v = \lambda f$  meaning that  $\lambda = \frac{v}{f}$ . From the fact that, in the case of constant velocity, distance is just speed times time, we have:

$$\lambda = (v + v_R) T'$$

$$T' = \frac{\lambda}{v + v_R} \quad (32-4)$$

for the period of the received oscillations. Using  $T' = \frac{1}{f'}$  and  $\lambda = \frac{v}{f}$  equation 32-4 can be written as:

$$\frac{1}{f'} = \frac{v/f}{v + v_R}$$

$$\frac{1}{f'} = \frac{v}{v + v_R} \frac{1}{f}$$

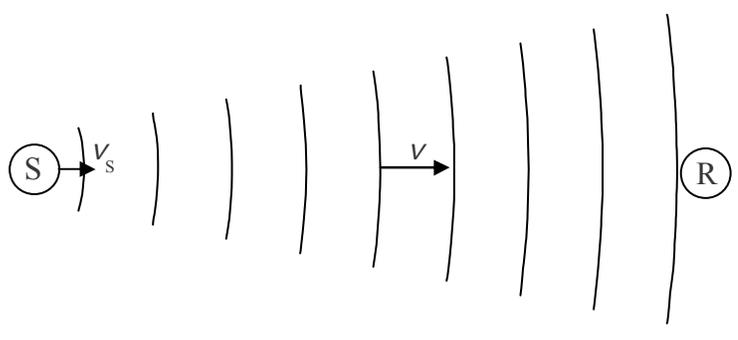
$$f' = \frac{v + v_R}{v} f \quad (\text{Receiver Approaching Source})$$

This equation states that the received frequency  $f'$  is a factor times the source frequency. The expression  $v + v_R$  is the speed at which the sound wave in air and the receiver are approaching each other. If the receiver is moving away from the source at speed  $v_R$ , the speed at which the

sound waves are “catching up with” the receiver is  $v - v_R$  and our expression for the received frequency becomes

$$f' = \frac{v - v_R}{v} f \quad (\text{Receiver Receding from Source})$$

Now consider the case in which the source is moving toward the receiver.



The source produces a crest which moves toward the receiver at the speed of sound. But the source moves along behind that crest so the next crest it produces is closer to the first crest than it would be if the source was at rest. This is true of all the subsequent crests as well. They are all closer together than they would be if the source was at rest. This means that the wavelength of the sound waves traveling in the direction of the source is reduced relative to what the wavelength would be if the source was at rest.

The distance  $d$  that the source travels toward the receiver in the time from the emission of one crest to the emission of the next crest, that is in period  $T$  of the source oscillations, is

$$d = v_s T$$

where  $v_s$  is the speed of the source. The wavelength is what the wavelength would be ( $\lambda$ ) if the source was at rest, minus the distance  $d = v_s T$  that the source travels in one period

$$\lambda' = \lambda - d$$

$$\lambda' = \lambda - v_s T \quad (32-5)$$

Now we'll use  $v = \lambda f$  solved for wavelength  $\lambda = \frac{v}{f}$  to eliminate the wavelengths and  $f = \frac{1}{T}$

solved for the period  $T = \frac{1}{f}$  to eliminate the period. With these substitutions, equation 32-5 becomes

$$\frac{v}{f'} = \frac{v}{f} - v_s \frac{1}{f}$$

$$\frac{v}{f'} = \frac{1}{f}(v - v_s)$$

$$\frac{f'}{v} = f \frac{1}{v - v_s}$$

$$f' = \frac{v}{v - v_s} f \quad (\text{Source Approaching Receiver})$$

If the source is moving away from the receiver, the sign in front of the speed of the source is reversed meaning that

$$f' = \frac{v}{v + v_s} f \quad (\text{Source Receding from Receiver})$$

The four expressions for the received frequency as a function of the source frequency are combined on your formula sheet where they are written as:

$$f' = \frac{v \pm v_R}{v \mp v_s} f \quad (32-6)$$

In solving a Doppler Effect problem, rather than copying this expression directly from your formula sheet, you need to be able to pick out the actual formula that you need. For instance, if the receiver is not moving relative to the air you should omit the  $\pm v_R$ . If the source is not moving relative to the air, you need to omit the  $\mp v_s$ .

To get the formula just right you need to recognize that when either the source is moving toward the receiver or the receiver is moving toward the source, the Doppler-shifted received frequency is higher (and you need to recognize that when either is moving away from the other, the Doppler-shifted received frequency is lower). You also need enough mathematical savvy to know which sign to choose to make the received frequency  $f'$  come out right.