

CHAPTER XXIV

PROBABILITY

229. Illustration. If a bag contains 3 white balls and 4 black balls, and 1 ball is taken out at random, what is the chance that the ball drawn will be white?

This question we may answer as follows: There are 7 balls in the bag and we are as likely to get one as another. Thus a ball may be drawn in 7 different ways. Of these 7 possible ways 3 will produce a white ball. Thus the chance that the ball drawn will be white is 3 to 7, or $\frac{3}{7}$. The chance that a black ball will be drawn is $\frac{4}{7}$.

230. General statement. It is plain that we may generalize this illustration as follows: If an event may happen in p ways and fail in q ways, each way being equally probable, the chance or probability that it will happen in one of the p ways is

$$\frac{p}{p+q} \quad (1)$$

The chance that it will fail is

$$\frac{q}{p+q} \quad (2)$$

The sum of the chances of the event's happening and failing is 1, as we observe by adding (1) and (2).

The odds in favor of the event are the ratio of the chance of happening to the chance of failure. In this case the odds in favor are

$$\frac{p}{q} \quad (3)$$

The odds against the event are $\frac{q}{p}$.

EXERCISES

1. If the chance of an event's happening is $\frac{1}{10}$, what are the odds in its favor?

Solution: By (1),
$$\frac{p}{p+q} = \frac{1}{10}.$$

Hence
$$10p = p + q,$$

or
$$9p = q,$$

or
$$\frac{p}{q} = \frac{1}{9},$$
 which by (3) are the odds in favor.

2. From a pack of 52 cards 3 are missing. What is the chance that they are all of one suit?

Solution: The number of combinations of 52 cards taken 3 at a time is $c_{52,3} = \frac{52 \cdot 51 \cdot 50}{1 \cdot 2 \cdot 3}$. This represents $p + q$. The number of combinations of the 13 cards of any one suit taken 3 at a time is $c_{13,3} = \frac{13 \cdot 12 \cdot 11}{1 \cdot 2 \cdot 3}$. This represents p .

Thus
$$\frac{p}{p+q} = \frac{\frac{13 \cdot 12 \cdot 11}{1 \cdot 2 \cdot 3}}{\frac{52 \cdot 51 \cdot 50}{1 \cdot 2 \cdot 3}} = \frac{13 \cdot 12 \cdot 11}{52 \cdot 51 \cdot 50} = \frac{11}{17 \cdot 25} = \frac{11}{425}.$$

3. What is the chance of throwing one and only one 6 in a single throw of two dice?

Solution: There are 36 possible ways for the two dice to fall. This represents $p + q$. Since a throw of two sixes is excluded there are 5 throws in which each die would be a 6, that is, 10 in all in which a 6 appears. This represents p .

Thus
$$\frac{p}{p+q} = \frac{10}{36} = \frac{5}{18}.$$

4. A bag contains 8 white and 12 black balls. What is the chance that a ball drawn shall be (a) white, (b) black?

5. A bag contains 4 red, 8 black, and 12 white balls. What is the chance that a ball drawn shall be (a) red, (b) white, (c) not black?

6. In the previous problem, if 3 balls are drawn, what is the chance that (a) all are black, (b) 2 red and 1 white?

7. What is the chance of throwing neither a 3 nor a 4 in a single throw of one die?

8. What is the chance in drawing a card from a pack that it be (a) an ace, (b) a diamond, (c) a face card?

9. Three cards are missing from a pack. What is the chance that they are (a) of one color, (b) face cards, (c) aces?
10. A coin is tossed twice. What is the chance that heads will fall once?
11. The chance that an event will happen is $\frac{3}{4}$. What are the odds in its favor?
12. The odds against the occurrence of an event are $\frac{3}{4}$. What is the chance of its happening?
13. What is the chance of throwing 10 with a single throw of two dice?
14. A squad of 10 men stand in line. What is the chance that A and B are next each other?
15. What is the chance that in a game of whist a player has 6 trumps?
16. What is the chance that in a game of whist a player holds 4 aces?